

# A Comparative Study of Cantilevered Ramp Injector with Standard k- $\epsilon$ and RNG k- $\epsilon$ Turbulence Models

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**Abstract**—In this study the combustion process in a SHCRAMJET using cantilevered compression and expansion type ramp injector having 60 of both compression and expansion angle is analyzed with Standard k- $\epsilon$  and RNG k- $\epsilon$  turbulence models using FLUENT software. After designing the model in GAMBIT, it is exported to FLEUNT software for analysis of combustion process with air inlet at Mach number 7 and hydrogen as the fuel with inlet Mach number 3.5. The results obtained with both the turbulence models are compared and they explain the variations of critical parameters like temperature, pressure, density, turbulent intensity and mass fraction of H<sub>2</sub>O formed and also the extent of combustion taking place in both the cases.

**Index Terms**—Hypersonic combustion, ramp injector, computational fluid dynamics, turbulent intensity.

## I. INTRODUCTION

Propulsion concepts such as the supersonic combustion ramjet (scramjet) and the shock-induced combustion ramjet (scramjet) utilize oxygen freely available in the atmosphere and thereby substantially reduce the weight penalty of on-board oxidizer tank used in rocket based systems. In the case of a hypersonic air-breather the challenge is increased due to the requirement of supersonic combustion. Flow velocities through the combustor on the order of thousands of meters per second provide the fuel and air with only a brief time to adequately combine. Contemporary mixing augmentation methods to address this issue have focused on fuel injection devices which promote axial vortices to enhance the mixing process.

Ramped fuel injectors employed as a means of fuel-air mixing enhancement has been the subject of a considerable amount of previous research. Interest in these fuel injectors was largely initiated by the National Aero-Space Plane (NASP) program in an effort to improve fuel/air mixing in scramjets. Northam [1] investigated a variety of wall-mounted injector ramps. Both swept and un-swept ramp injectors were studied in various duct configurations. Emphasis was placed on near parallel injection for thrust recovery at high vehicle flight Mach numbers. Consequently, fuel was injected from ramps at an angle of 10.3° to the combustor wall. Fuel at  $M_f = 1.7$  was injected into a  $M_a = 2$  airflow. The injector design incorporated reflected shock waves intersecting the injected fuel to enhance mixing. Analysis of results comprised largely of shadowgraph flow

visualization and combustion efficiency calculations. Results found the swept ramp injector to have generally superior performance over the un-swept ramp. A combination of ramp and subsequent downstream perpendicular injection was found to improve combustion efficiency only in the case of un-swept injectors. An experimental effort was conducted by Hartfield [2] considering very similar ramp injector. Their work revealed highly three-dimensional flow fields which "dramatically illustrate the domination of the mixing process by stream wise vorticity generated by the ramp". Experiments consisted of non-reacting flow with seeded air injected into free stream air for free stream conditions of Mach 2 and Mach 2.9. A primary objective of the study was the determination of the influence of free stream Mach number on injector performance for a given injector geometry. A laser-induced iodine fluorescence technique was employed to collect temperature and injectant concentration data. Analysis of global mixing performance was limited to a parameter reflecting the percentage of duct area mixed to within static flammability limits. It was found that the injectant mixed faster at lower free stream Mach numbers. Comprehensive flow field visualization was presented clearly delimiting the Vortical flow structures. A reacting Navier-Stokes code was utilized by Rigging [3] to solve both swept and unswept ramp configurations with fuel injected at Mach 1.7 through circular orifices from ramp injectors. The code employed a two step finite-chemistry model together with the Baldwin-Lomax turbulence model. Both reacting and non-reacting cases were considered with laminar flow calculated for non-reacting cases. The numerical solutions afforded a more thorough performance analysis including measures of circulation, fuel concentrations, mixing efficiency, and total pressure recovery and, in the case of reacting flow, combustion efficiency. Basic gridding techniques and boundary condition treatments were used and grid convergence issues were not addressed. Results showed substantially higher mixing performance as well as flow losses for the swept configuration over the unswept ramp. The study concluded that vorticity increased fuel mixing and that near-field mixing was controlled by large-scale vortices while far-field mixing was controlled by smaller scale turbulent diffusive processes.

A significantly improved injector design was proposed by Marble [4] et al. who introduced the "contoured wall fuel injector". The ramp injector design was integrated with the combustor wall which allowed for shock structures promoting baroclinic torque generated vorticity. A combined computational and experimental effort focused on the demonstration of enhanced mixing through the generation of stream wise vorticity and its use for hypersonic

propulsion. It was determined that the characteristic mixing times were fast enough for scramjet applications. While limited quantitative analysis and no comparison with other designs was presented, a ground breaking proof of concept was shown. The investigation concluded that the ramp injector under consideration "can lead to rapid enhancement of the mixing process". A further conclusion was that a mechanism to destabilize the large vortices must be sought to ensure complete mixing. A modified design to Marble's injector was studied by Davis[5]. The fuel injector embodied the elements of the contoured wall fuel injector but was more modular since it could be mounted on a flush combustor wall. The study focused on jet penetration and mixing behaviour under a variety of different operating conditions. A swept and unswept injector configuration was experimentally tested. Unfortunately, quantification of mixing performance was minimal and comparison with other injectors was absent. The superior vortex generating ability of the swept configuration over the unswept was established. A thorough investigation of the injector design advanced by Marble was undertaken by Waitz . A concurrent experimental and numerical effort was undertaken to study Mach 1.7 helium (used to simulate hydrogen) injected into a Mach 6 airstream. Several parametric dependencies were investigated including: injector spacing, ramp geometry, boundary layer effects and injectant/free stream velocity and pressure ratios. A detailed description of flow fields and flow phenomena were presented. The work demonstrated that the induced vorticity coalesced into a counter-rotating pair of vortices promoting helium migration up into the main stream. The two main sources of vorticity, baroclinic torque and cross-streams shear, were identified and characterized. It concluded that shock-impingement produced effective mixing by deposition of baroclinic torque at the fuel air interface while cross-stream shear induced vorticity can be less effective due to vortices generated remote from the fuel air interface. Flow visualization was employed to identify salient flow features. Excellent comparisons of experimental and computational results were presented along with comprehensive mixing performance and loss analysis. The suitability of injector design to scramjet applications was addressed and it was concluded that the injector design in question was a feasible candidate for mixing enhancement. An interesting approach to improve fuel/air mixing enhancement is through the use of various nozzle geometries used to inject fuel from ramp injector.

Haimovitch experimentally investigated six different injector-nozzle inserts to precondition the fuel flow. The main objective was to determine the influence of the resultant fuel jet on the vortical flow field induced by the ramp injectors. Seeded air at Mach 1.63 was used to simulate fuel injected into a Mach 2 main stream. Mie scattering visualization revealed a minor difference in the mixing performance between the candidate injectors. More comprehensive computational results were provided by Eklund[6] who studied Mixing in the context of a reacting flow field the Navier-Stokes equations were solved with a finite-rate chemical kinetics model for H<sub>2</sub>-air reaction together with an algebraic eddy viscosity turbulence model .Two configurations: swept compression and swept

expansion ramp injectors, were used to inject Mach 1.7 fuel at 10<sup>0</sup> with respect to the main flow. A major conclusion of the investigation was that mixing was significantly reduced by combustion. A reduction of up to 25% in mixing efficiency was observed for the reacting case. Riggins and Via [7] furthered investigation of generic swept and unswept ramp injectors with a more refined numerical model. Larger computational grids were employed simulating laminar and turbulent mixing. Insightful analysis of results underscored the dominant role of turbulence in the far-field, although turbulence modelling issues were not fully addressed. Comparison with high-enthalpy experimental results determined that CFD is a valuable engineering tool for injector design. More sophisticated numerical modelling was applied to ramp injectors by Lee[8]. The contoured wall injector design of Marble was investigated to determine the mixing characteristics in the presence of combustion. A numerical algorithm employing the three-dimensional Navier-Stokes equations coupled with a chemical reaction model and a K-epsilon turbulence model was used for the study. Freestream air conditions were held constant while changing initial fuel pressure and density. The study concluded that changes in fuel density had a significant impact on mixing and combustion performance while pressure changes had little effect. It further asserted that the mixing process has a strong influence on combustion, whereas the combustion process does not have any significant effect on the mixing process. The results suggest that the mixing process may be decoupled from the combustion process with only minor differences in performance trends. A more intricate ramp injector configuration was studied by Baurle[9] in a combined experimental and computational investigation. Ramp injectors were mounted on opposite sides of the combustor in an indigested fashion with four fuel injection ports located at the base of each ramp. The nozzles in the base of the ramps were angled with respect to the combustor wall and a yaw angle was also introduced. The injector nozzle flow was included as part of the computational domain which consisted of a remarkable 13.5 million grid nodes. A more intricate ramp injector configuration was studied by Baurle in a combined experimental and computational investigation. Ramp injectors were mounted on opposite sides of the combustor in an indigested fashion with four fuel injection ports located at the base of each ramp. The nozzles in the base of the ramps were angled with respect to the combustor wall and a yaw angle was also introduced. The injector nozzle flow was included as part of the computational domain which consisted of a remarkable 13.5 million grid nodes. J. Schumacher [10] studied the Numerical Simulation of Cantilevered Ramp Injector Flow Fields for Hyper velocity Fuel-Air Mixing Enhancement. Dudebout, R., Sislian, J. P., and Oppitz, R.[12] studied Hypersonic air-breathing propulsion using shock-induced combustion ramjets using 2D geometries with planar and axisymmetric configurations, as well as external and mixed-compression configurations. The lower-upper symmetric Gauss-Seidel scheme, combined with a symmetric shock-capturing total variation diminishing scheme, were used to solve the Euler equations, with non equilibrium chemical reactions.

II. METHODOLOGY

In this part we aim towards the formulation of the problem and realization of constraints and pre and post defining the problem. The main objectives in this stage were :-

To initiate combustion across the air-fuel(H<sub>2</sub>) mixture with air inlet at Mach no. 7 and H<sub>2</sub> from the ramp injector at Mach no. 3.5.

To find the temperature distribution and the other critical parameters and quantities across the air-fuel mixture with standard k-ε and RNG k-ε turbulence models.

The 2D modeling scheme was adopted in GAMBIT. In fig.1 as shown below, 1,2,3 represent fuel mass-flow inlet, air mass-flow inlet and pressure outlet respectively and the remaining edges were designated as wall. It was analyzed using FLUENT.

For purpose of defining the physical model we used the following values :

- Air inlet edge = 60 mm.
- Fuel inlet edge = 20 mm.
- Outlet edge = 100 mm.
- Length of the ramp = 320 mm.
- Length of the combustion chamber = 600 mm.
- Angle of expansion (α<sub>e</sub>) = 6°.
- Angle of compression (α<sub>c</sub>) = 6°.

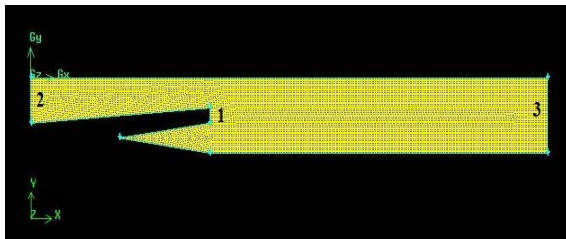


Fig.1 Meshed 2D model.

TABLE I INLET CONDITIONS

	Mach number	P (Pa)	T (K)	ρ (kg/m <sup>3</sup> )	Mass flow rate (kg/s)
Air	7.0	16500	900	0.0636	0.945
Fuel	3.5	16500	240	0.1160	0.027

TABLE II MASS FRACTIONS AT INLET

	Hydrogen	Oxygen
Air inlet	0.029	0.228
Fuel inlet	1.000	0.000

A. Problem Setting

The .msh file obtained from the GAMBIT was exported to FLUENT for subsequent analysis. The .msh file was read using FLUENT and subsequently its grid checking was done, the grid was checked with no error and the formation of one default interior.

The following models were selected:

- The density based solver,
- Energy equation,
- Standard k-ε model,
- RNG k-ε model,
- Eddy-dissipation criterion in the species transport section.

B. Materials and Material Properties

Mixture as ideal gas,  
Cp, k (conductivity) and ρ (density) vary according to ideal gas mixing law.

Air-H<sub>2</sub> as the fuel mixture

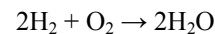
Wall material as aluminium.

C. Input Data

The wall temperature was taken as 800 K. The A/F (air fuel ratio) is 34.47.

D. Reaction Model

The instantaneous reaction model assumes that a single chemical reaction occurs and proceeds instantaneously to completion. The reaction used for the shramjet was the hydrogen-water reaction:



III. RESULTS AND DISCUSSIONS

A. Stanadard k-ε Turbulence Model

1) Variations of temperature across the air-fuel mixture

At the zones where combustion takes place the maximum static temperature reached is 2740 K as can be seen from fig.2 and the maximum total temperature reached is 8040 K as can be seen from fig. 3.

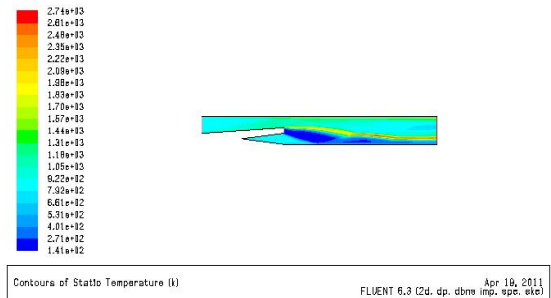


Fig. 2 Contours of static temperature

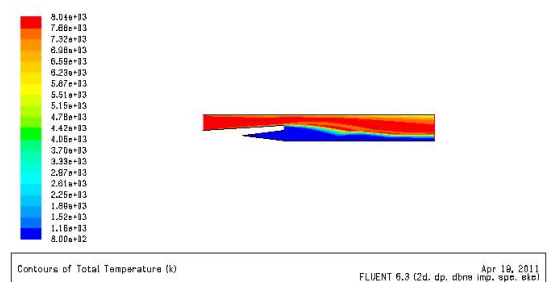


Fig. 3 Contour of total temperature

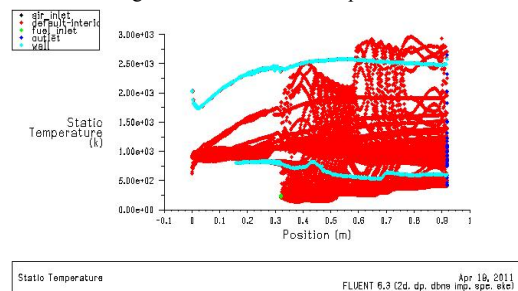


Fig. 4 Static temperature vesus position.

The fig. 4 shows that the air inlet temperature is 900 K and

the wall temperature is 800 K. The static temperature of air goes on increasing till 0.3m where combustion is initiated and there is an exponential rise in static temperature. The peak temperature is found to be 3000 K approximately.

2) Variation of turbulent intensity across the air-fuel mixture

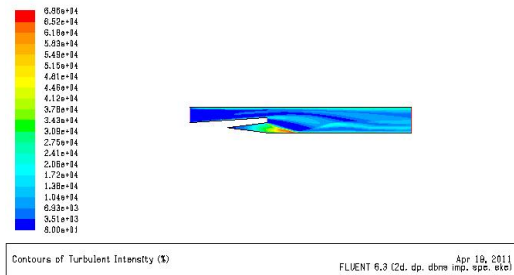


Fig. 5 Contours of turbulent intensity

The fig. 5 shows that the turbulent intensity in the immediate vicinity of the ramp injector is stupendously high. In the region just below the ramp the red and yellow colored zone shows an increase in the turbulent intensity in the order of 69600% w.r.t. the turbulent intensity at the air inlet. A very high turbulent intensity represents a superior air-fuel mixing.

3) Variation of mass fraction of H<sub>2</sub>O

The fig. 6 shows the variation of mass fraction of H<sub>2</sub>O. The formation of H<sub>2</sub>O is a strong indicator of the extent of combustion. The maximum mass fraction of H<sub>2</sub>O being generated is 0.975.

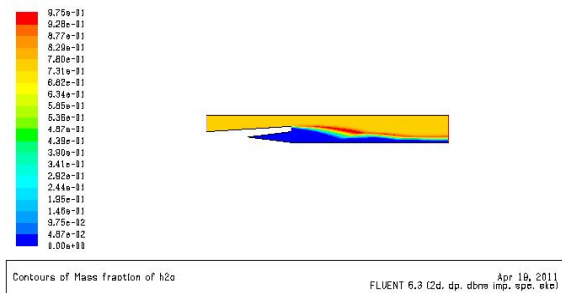


Fig. 6 Contours of mass fraction of H<sub>2</sub>O

4) Variation of Mass Fraction of O<sub>2</sub>

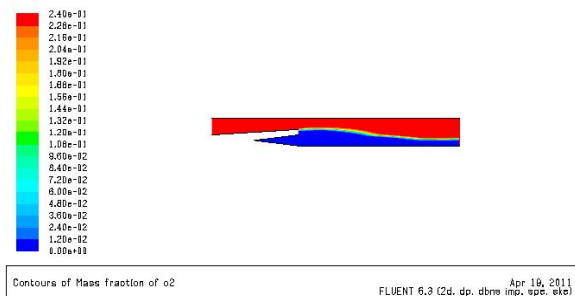


Fig. 7 Contours of mass fraction of O<sub>2</sub>

The fig. 7 shows the mass fraction of O<sub>2</sub>. Oxygen is consumed at the zones of combustion evident from its lower value of mass fraction in those zones.

5) Variation of Density

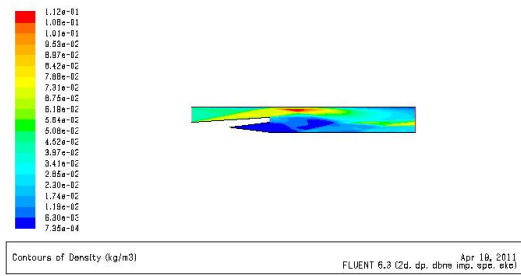


Fig. 8 Contours of density

The yellow and red region in the fig. 8 shows dramatic increase in the density. It is due to the fact that a shock is produced due to the compression angle of the ramp which also augments the air-fuel mixing.

6) Variation of Pressure

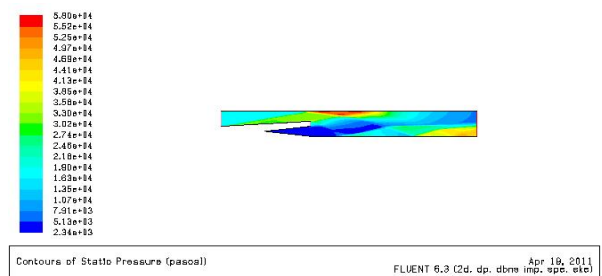


Fig. 9 Contours of static pressure

The fig. 9 shows the variation of static pressure across the air-fuel mixture. The maximum value of static pressure is 58000 Pa.

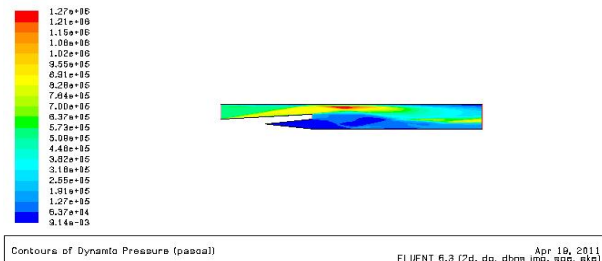


Fig. 10 Contours of dynamic pressure.

The fig. 10 shows the variation of dynamic pressure. The maximum value of dynamic pressure is 1270000 Pa and the minimum value is 63700 Pa.

B. RNG k-ε Turbulence Model

1) Variations of Temperature Across the Air-fuel Mixture

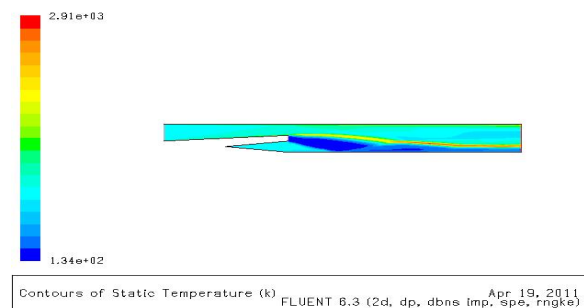


Fig. 11 Contours of Static Temperature

At the zones where combustion takes place the maximum

static temperature reached is 3430 K as can be seen from the red zone in fig.11 and the maximum total temperature reached is 8010 K as can be seen from fig. 12.

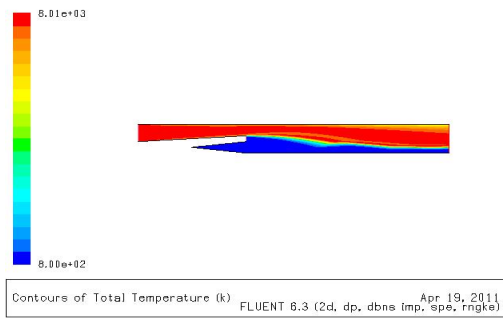


Fig. 12 Contours of Total Temperature

The fig. 13 shows that the air inlet temperature is 900 K. The static temperature of air goes on increasing till 0.3m where combustion is initiated and there is a exponential rise in static temperature. The peak temperature is found to be 3000 K approximately.

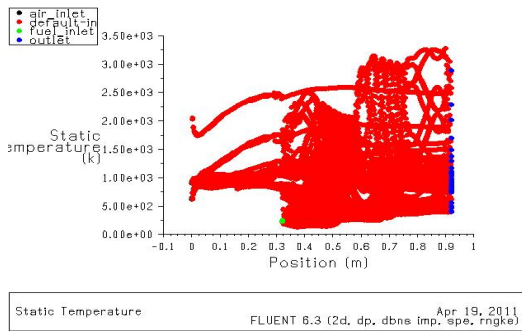


Fig. 13. Static Temperature versus Position.

### 2) Variation of Turbulent Intensity Across the Air-fuel Mixture

The fig. 14 shows that the turbulent intensity in the immediate vicinity of the ramp injector is stupendously high. In the region just below the ramp the red and yellow colored zone shows an increase in the turbulent intensity in the order of 62900% w.r.t. the turbulent intensity at the air inlet. A very high turbulent intensity represents a superior air-fuel mixing.

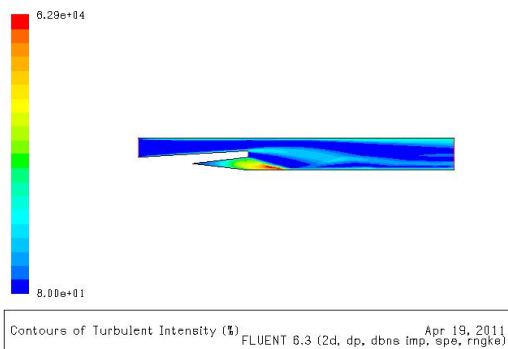


Fig. 14 Contours of Turbulent Intensity.

### 3) Variation of mass fraction of H<sub>2</sub>O

The fig. 15 shows the variation of mass fraction of H<sub>2</sub>O. The formation of H<sub>2</sub>O is a strong indicator of the extent of combustion. The maximum mass fraction of H<sub>2</sub>O being generated is 0.974. This is shown by the red zone.

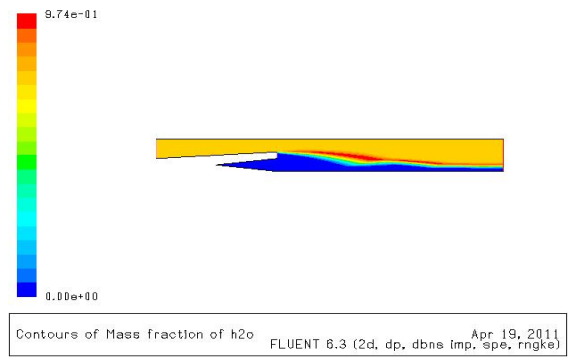


Fig. 15 Contours of mass fraction of H<sub>2</sub>O.

### 4) Variation of Mass Fraction of O<sub>2</sub>

The fig. 16 shows the mass fraction of O<sub>2</sub>. Oxygen is consumed at the zones of combustion evident from its lower value of mass fraction (shown by the green and yellow zone) in those zones.

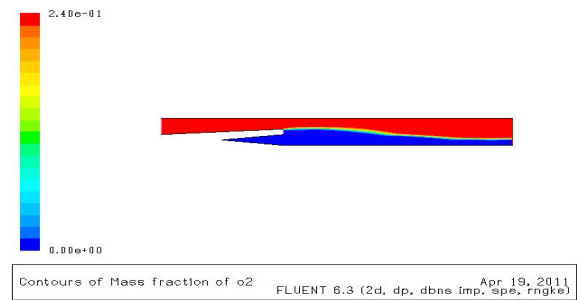


Fig. 16. Contours of mass fraction of O<sub>2</sub>.

### 5) Variation of Density

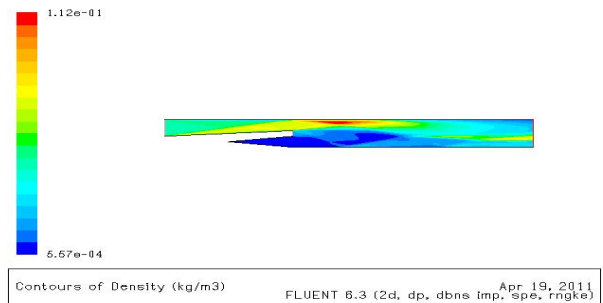


Fig. 17 Contours of density

The yellow and red region in the fig. 17 shows dramatic increase in the density. It is due to the fact that a shock is produced due to the compression angle of the ramp which also augments the air-fuel mixing.

### 6) Variation of Pressure

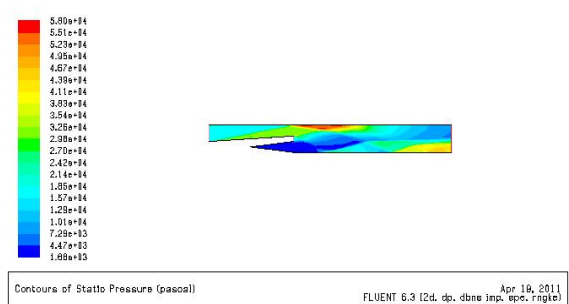


Fig. 18 Contours of Static Pressure.

The fig. 18 shows the variation of static pressure across the air-fuel mixture. The maximum value of static pressure is

58000 Pa. The static pressure goes on increasing from the zone of initiation of combustion to the pressure outlet plane.

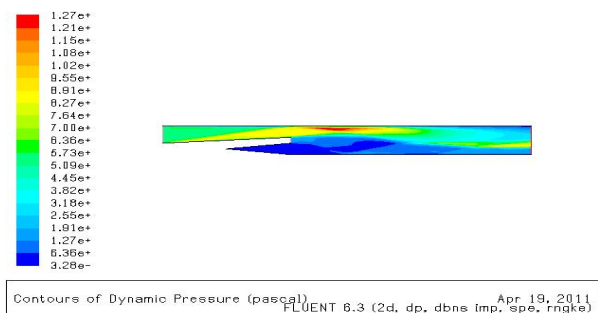


Fig. 19 Contours of Dynamic Pressure

The fig. 19 shows the variation of dynamic pressure. The maximum value of dynamic pressure is 1270000 Pa and the minimum value is 328000 Pa. The dynamic pressure also goes on increasing from the zone of initiation of combustion to the plane of pressure outlet.

TABLE III COMPARATIVE: ANALYSIS

Types	Maximum Temperature (K)	Maximum Mass fraction of H <sub>2</sub> O
Standard k-ε model	2.74e+03	9.75e-01
RNG k-ε model	2.91e+03	9.74e-01

#### IV. CONCLUSION

The above analysis reaches to the following conclusions mentioned below:

- The standard k-ε model is a simple two-equation model which is stable and numerically robust. It provides considerably accurate solutions for turbulent flow only. It can't take into account the laminar flow regions for study. The maximum temperature reached in the k-ε model is 2740K and the mass fraction of H<sub>2</sub>O is 97.4%. The occurrence of high turbulent intensity represents a high air-fuel mixing.
- The RNG k-ε model was developed using Re-Normalization Group (RNG) to renormalize the Navier-Stokes equations, to account for the effects of smaller scales of motion. The maximum temperature reached is 2910K and the mass fraction of H<sub>2</sub>O is 97.5%. It shows a better combustion and a superior air-fuel mixing.
- The RNG k-ε model is a superior one with high temperature of combustion and superior air-fuel mixing. So it is the best turbulence model among the three. This is because the RNG approach, which is a mathematical technique that can be used to derive a turbulence model similar to the k-epsilon, results in a modified form of the epsilon equation which attempts to account for the different scales of motion through changes to the production term.
- The turbulent intensity with the RNG k-ε turbulence model is high in the immediate vicinity of the ramp injector indicating a superior air-fuel mixing. It is of the order of 62900% with respect to the turbulent intensity at the inlet. A very high turbulent intensity indicates a superior air-fuel mixing. An enhanced air-fuel mixing has been possible due to the geometry of the cantilevered ramp injector. The geometry of the ramp injector produces a shock due to which a highly turbulent environment is generated thus

enhancing air-fuel mixing and thus superior quality of combustion.

- The high value of mass fraction of H<sub>2</sub>O formed with the standard k-ε turbulence model indicates an efficient combustion process. The maximum value of mass fraction of water formed is 0.975 which indicates nearly complete combustion of the air-fuel mixture in the zones where it is formed.
- The sudden rise in density observed near the tip of the ramp injector indicates the generation of shocks which help in superior air-fuel mixing. Superior air-fuel mixing resulting in better quality of combustion and thus better performance.

#### V. ACKNOWLEDGEMENT

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